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PREDICTING FATIGUE LIFE OF SHORT FIBER REINFORCED VISCOELASTIC COMPOSITES

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Анотоція. *Моделюється процес довготривалого руйнування в'язкоупружних матеріалів при загальних тривісних умовах навантаження. Метою є оцінка залежності еволюції локальної концентрації напружень у часі від тривісності напружень і параметра Лоде. Визначальні рівняння матеріалу описуються з використанням похідних дробового порядку. Співвідношення напруження-деформація сформульовані на основі узагальнення моделі типу Работнова, яка описує цикли гистерезису зі зростаючими напруженнями при зростанні кількості циклів навантаження. Встановлено критерії для визначення числа циклів, які можуть викликати руйнуваання втоми для зразків при заданих амплітудах і частотах. В складних умовах навантаження розрізняються гидростатичні навантаження і різні стани напружень зсуву. Розглянуто суто тривісні макроскопічні умови навантаження з різними головними напруженнями.Пропонується узагальнена модель в'язкопружнсті із внутрішнім параметром пошкодження. Локальні та макроскопічні визначальні співвідношення виражені як спадкові інтеграли. Оскільки об'ємна концентрація фаз лишається незмінною у часі, то використовується принцип Вольтерра та перетворення Карсона. Для розрахунків використано програми Fortran 90 з бібліотеки NAG-Fortran. Концентрація напружень навколо включень змодельована, як приклад, у трикомпонентному композитному матеріалі на основі матриці з епоксидної смоли. Запропонована нелінійна модель, може бути корисною для прогнозу довготривалої міцності і неруйнівних засобів контролю стану композитів.*

Keywords: *composite material, fatigue life, visco-elastic, stress concentration.*

The composite materials used in progressive technics structures can experience fatigue damage and failure due to the repeated loads. Hence, the models that stimulate the response of composites under cyclic loads are necessary to design structures of long term strength. Theoretical estimation of remaining lifetimes and residual strength is an important modern problem of solid mechanics. The response of composite structures under fatigue loading is a rather new problem that has led to the development of a number of fatigue prediction models. The focus of this paper is on the strength degradation effects, continuum damage mechanics approach, and micromechanics models capabilities [1-3].

A commonly used approach in fatigue life predictions is to use stress versus life, known as $S-N$ curves. The constant amplitude cyclic loads are characterized by the mean stress level σ^m and the amplitude σ^a of the stress variations around the mean. This is alternatively expressed in terms of the maximum stress and the stress ratio or R-ratio. The situation is more complex in the case of heterogeneous media, strong stress triaxiality, and rheology time presence. For the analysis of creep fatigue problems in the framework of the quasi-linear viscoelasticity model, we use the correspondence principle, which is different from that used in the linear theory [4]. In this case, there is no assumption of an analogy between the defining relations of nonlinear elasticity and nonlinear viscoelasticity. Let t be the time, $\mathbf{x}$, $\boldsymbol{\sigma}(\mathbf{x},t)$, $\mathbf{e}(\mathbf{x},t)$ and $\mathbf{u}(\mathbf{x},t)$ be the position, the current stress, the current strain, and the current displacement in three-dimensional case, respectively. We assume that the viscoelastic material possesses instantaneous elastic response $\boldsymbol{\sigma}^{el}(\mathbf{x},t)$, $\mathbf{e}^{el}(\mathbf{x},t)$, $\mathbf{u}^{el}(\mathbf{x},t)$. The model requires that the loading curves and the unloading curves must fall in the same curve, and the stress and the strain must return to the origin simultaneously. It follows that there exists a strain energy function $W(\mathbf{e},\mathbf{x},t)$ with the property that

$$W = W(\mathbf{e}^{el},\mathbf{x},t), \quad \boldsymbol{\sigma}^{el} = \frac{\partial W}{\partial \mathbf{e}^{el}}. \tag{1}$$

This equation defines the nonlinear elastic constitutive relations. To formulate the correspondence principles, we write down the constitutive equations of quasi-linear viscoelasticity between the current stress $\boldsymbol{\sigma}(t)$ and the instantaneous (elastic) stress $\boldsymbol{\sigma}^{el}(t)$

$$\boldsymbol{\sigma}^{el}(t) = \mathbf{g} \star d\boldsymbol{\sigma} = \int_{-\infty}^{t} \mathbf{g}(t-t_1) d\boldsymbol{\sigma}(t_1), \quad \mathbf{g} = \mathbf{J}(t) / \mathbf{J}(0)$$

$$\boldsymbol{\sigma}^{el}(t) = \boldsymbol{\varphi}[\mathbf{e}(t)] = \mathbf{g} \star d\boldsymbol{\sigma} = \boldsymbol{\sigma} \star d\mathbf{g}, \quad \boldsymbol{\sigma} = \mathbf{r} \star d\boldsymbol{\sigma}^{el}. \tag{2}$$

And constitutive relations for creep

$$\mathbf{e}^{el}(t) = \boldsymbol{\psi}[\boldsymbol{\sigma}(t)] = \mathbf{h} \star d\mathbf{e} = \mathbf{e} \star d\mathbf{h} = \mathbf{h} \star d\mathbf{e}, \quad \mathbf{h} = \mathbf{E}(t) / \mathbf{E}(0)$$

$$\boldsymbol{\sigma}^{el}(t) = \boldsymbol{\varphi}[\mathbf{e}(t)] = \mathbf{g} \star d\boldsymbol{\sigma} = \boldsymbol{\sigma} \star d\mathbf{g}, \quad \boldsymbol{\sigma} = \mathbf{r} \star d\boldsymbol{\sigma}^{el}. \tag{3}$$

Quasi-linear viscoelasticity allows generalizing the classical approaches in mechanics of composites [1]. We use here the enhanced viscoelastic model with internal parameter of stored damage D [3]. The local and overall constitutive relations between the infinitesimal strain $\mathbf{e}(\mathbf{x},t)$ and the Cauchy stress $\boldsymbol{\sigma}(\mathbf{x},t)$ fields can be expressed as hereditary integrals. At the micro-scale of individual r constituents these are presented by [5]

$$\mathbf{e}(\mathbf{x},t) = (\mathbf{q}_r \star \mathbf{e}^{el})(\mathbf{x},t), \quad \mathbf{x} \in v_r. \tag{4}$$

Space coordinate $\mathbf{x}$ denotes a material point within any phase r of the composite and $\star$ stands for the Stieltjes convolution product. Similarly, the macroscopic or effective constitutive relations can be written as

$$\langle \mathbf{e} \rangle (t) = (\tilde{\mathbf{q}} \star \left\langle \frac{\partial U}{\partial \boldsymbol{\sigma}} \right\rangle)(t) \tag{5}$$

Here $\langle \mathbf{e} \rangle (t)$ and $\langle \boldsymbol{\sigma} \rangle (t)$ are the macroscopic, or averaged, strain and stress, the angle brackets denote spatial averaging over a representative volume element of the material. Four order tensors $\mathbf{q}_r(t)$ and $\tilde{\mathbf{q}}(t)$ are the local in phase r and effective creep reduced functions of the composite, respectively.

Basic concepts of damage mechanics were formulated at the theoretical level [1, 3, 5], in particular through thermodynamic formalism. Note that the nonlinear response of composite could be enhanced by strength reduction damage [6]. The strain equivalence hypothesis, which states that any deformation behavior, whether uniaxial or multi-axial, of a damaged material is represented by the constitutive laws of the virgin material in which the usual stress $\sigma(t)$ is replaced by the so-called effective stress $\tilde{\sigma}(t)$, which enables the definition of an effective stress

$$\tilde{\sigma}(t) = \sigma(t)(1-D)^{-1}. \tag{6}$$

Here $\tilde{\sigma}(t)$ is defined as the stress in the effective (undamaged) state. Therefore, $\tilde{\sigma}(t)$ has been termed the effective stress. Thus, in the presence of damage $(0 < D < 1)$, the effective area is reduced by a factor of $(1 - \mathrm{D})$, while the effective stress is increased by the same factor, so that the force $\tilde{\sigma}\bar{A} = \sigma A$ is preserved. In our model, the viscoelastic strain energy function $W(t)$ is coupled with damage parameter D. The expression of $W(t)$ is defined as [2]

$$2W(\mathbf{e},t) = (1-D(t)) \int_{-\infty}^{t} \int_{-\infty}^{t} \frac{\partial \mathbf{e}(t_1)}{\partial t_1} \mathbf{E}(2t - t_1 - t_2) \frac{\partial \mathbf{e}(t_2)}{\partial t_2} dt_1 dt_2, \tag{7}$$

$$Y = -\partial W / \partial D$$

where $.\mathbf{E}(t).$ is the relaxation tensor. The internal scalar variable D models the damage, which is assumed to be isotropic and varies between 0 for undamaged material and 1 under complete failure. The thermodynamic force associated $\mathbf{e}(t)$ with D is denoted. The constitutive equation may be written in the compliance formulation to describe creep phenomena

$$2(1-D(t))U(\mathbf{s},t)=\int_{-\infty}^{t}\int_{-\infty}^{t}\frac{\partial\boldsymbol{\sigma}(t_2)}{\partial t_2}\mathbf{J}(2t-t_1-t_2)\frac{\partial\boldsymbol{\sigma}(t_1)}{\partial t_1}dt_1dt_2. \qquad (1)$$

According to (5) in quasi-linear viscoelasticity, for the proposed viscoelastic model coupled with damage the expression of stress is written as

$$\boldsymbol{\sigma}(t)=(1-D(t))\int_{-\infty}^{t}\mathbf{h}(t-t_1)\frac{\partial W(\mathbf{e},t_1)}{\partial t_1}dt_1. \qquad (2)$$

The stress $\boldsymbol{\sigma}(t)$ is thus related to the damage variable $D(t)$ and to the whole history of viscoelastic strains throw the energy $W(\mathbf{e},t)$ via Boltzmann's hereditary integral. Note that the constant volume concentration of phases remains unchanged after transforming from the time domain to the Carson domain. The Fortran95 programs from NAG-Fortran library we use for numerical analysis required. Statistical averaging of expressions is performed to define the mean deformation of short inclusions randomly oriented in volume. The result is that overall response of such a composite is isotropic [7]. Stress concentration near inclusions and overall creep response are modeled in the three-component metal matrix composite with aluminum viscoelastic matrix [1].

In our work, we use Hashin's [2] failure criteria to determine the fiber and matrix failures in a multicomponent composite. Equations that summarize the failure envelopes for fiber and matrix failure are obtained from Hashin's criteria. In particular, short fibers and matrix failure in tension will be

$$\left(\frac{\sigma_{11}}{X_T}\right)^2+\frac{\sigma_{12}+\sigma_{13}}{S_{12}^2}=1, \quad \frac{(\sigma_{22}+\sigma_{33})}{Y_T^2}+\frac{\sigma_{23}^2-\sigma_{22}\sigma_{33}}{S_{23}^2}+\frac{\sigma_{12}^2+\sigma_{13}^2}{S_{12}^2}=1. \qquad (9)$$

In equations (9), X_T and X_C are the longitudinal tensile and compressive strengths, Y_T and Y_C are the transverse tensile and compressive strengths, S_{12} is the in-plane shear strength, and S_{23} is the out of plane shear strength. An instantaneous matrix stiffness degradation scheme is used for the progressive failure when matrix or fiber failure is detected. We evaluate here the residual stiffness of the representative volume following failure in each mode [2]. In other words, the fatigue model used here is based on stiffness and strength reduction directly applied to the engineering stiffness constants and strengths that are RVE properties. To quantify and visualize the level of damage, a measure of the relative reduction in the stiffness/strength parameter due to damage D_P is calculated using equation (9)

$$D_P=1-\frac{P_r}{P_{init}}. \qquad (10)$$

The non-linear cumulative damage rule for isotropic viscoelastic composite materials is used here. Scalar damage variable $D(t)$ evolves with the number of cycles. The evolution of damage is governed by increment methods [5]

$$\int_{D_{(k-1)}}^{D_k}dD=\int_0^N[1-(1-D)^{1+\beta_f}]^{\alpha_f}\left(\frac{\sigma_k}{1-D}\right)^{\beta_f}dN.$$

N is the number of cycles at the current stress state σ_k, D_k and D_{k-1} are the amount of damage after the current, and previous cycles, respectively, β_f is a material parameter, and α_f is a function of the current triaxial stress state [6].

The fatigue life of composites is evidently connected with stress concentration on the interphase surfaces. To present the formulation of the general interface model we introduce the following normal $\boldsymbol{\nu}$ and tangent $\boldsymbol{\eta}$ projection tensors of second order

$$\boldsymbol{\nu}=\mathbf{n}\otimes\mathbf{n}; \quad \boldsymbol{\eta}=\mathbf{1}-\boldsymbol{\nu}. \qquad (11)$$

Symbol $\mathbf{1}$ is the 3D second-order identity tensor. Let us construct further the normal $\mathbf{N}$ and tangent $\mathbf{T}$ projection tensors of fourth order by

$$\mathbf{N} = \mathbf{I} - \mathbf{T}; \quad \mathbf{T} = \boldsymbol{\eta} \otimes \boldsymbol{\eta}, \tag{12}$$

$\mathbf{I}$ is the fourth-order identity tensor for the space of second-order symmetric tensors. In fact, $\mathbf{T}$ and $\mathbf{N}$ correspond to the exterior and interior projection operators of Hill [1]. Next, we write

$$N_{ijkl} = \frac{1}{2}(\delta_{ik}\nu_{jl} + \delta_{jk}\nu_{il} + \delta_{il}\nu_{jk} + \delta_{jl}\nu_{ik}) - \nu_{ij}\nu_{kl}, \quad T_{ijkl} = \frac{1}{2}(\eta_{ik}\eta_{jl} + \eta_{jk}\eta_{il})$$

$$\Gamma_{ijkl} = (2\mu)^{-1}\left(N_{ijkl} - \frac{\nu}{1-\nu}n_i n_j n_k n_l\right); \quad \Pi_{ijkl} = 2\mu\left(T_{ijkl} + \frac{\nu}{1-\nu}\eta_{ij}\eta_{kl}\right),$$

$$\boldsymbol{\Gamma}(\mathbf{n}) = (\mathbf{N\ E\ N})^{-1}, \quad \boldsymbol{\Pi}(\mathbf{n}) = (\mathbf{T\ J\ T})^{-1}. \tag{13}$$

It can be shown that $\boldsymbol{\Gamma}$ is given by [1]

$$\boldsymbol{\Gamma}(\mathbf{n}) = \frac{1}{2}(\mathbf{G}(\mathbf{n}) \otimes \mathbf{N} + \mathbf{N} \otimes \mathbf{G}(\mathbf{n})), \tag{14}$$

where the second-order tensor $\mathbf{G}$ is calculated by $\mathbf{G} = \mathbf{Q}^{-1}$, $\mathbf{Q} = \mathbf{n\,E\,n}$. In addition, the tensors $\boldsymbol{\Gamma}(\mathbf{n})$, $\boldsymbol{\Pi}(\mathbf{n})$, relaxation function $\mathbf{E}(t)$ and creep function $\mathbf{J}(t)$ are connected by the identity $\mathbf{J\Pi} + \mathbf{\Gamma E} = \mathbf{I}$.

Nonlinearity of metal matrix is connected with instant elastic properties and stored in time micro-defects. Some numerical examples were analyzed. Internal stress concentration and fatigue life was modeled for metal matrix composite with constituents: nonlinear visco-elastic Al8091 matrix ($\lambda = 44.93GPa$, $\mu = 31.0\ GPa$, $\nu_3 = -435GPa$), and nonlinear elastic boron and SiC short fibers. Properties of fibers are presented in Table 1 in [8]. Our approach realized here is taking into account not only mean field stress state [1] but local stresses near short fiber in viscoelastic matrix [8] due to cyclic external loading. As a part of conclusions it should be noted that results of fatigue life prediction with the model proposed are in an acceptable correlation with known from literature experimental data. Due to multi-parameter nature of process and used approach, it is needed to continue this work, especially in the sense of identification material constants.

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